Lecture 8: Diversification

ECON435: Financial Markets and the Macroeconomy

Anton Korinek

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Review: Dealing with Risk

- Utility function $U = E(r) - ½A\sigma^2$
- Asset allocation:
  - Capital allocation line (CAL)
  - Optimal portfolio choice

Principle of Diversification

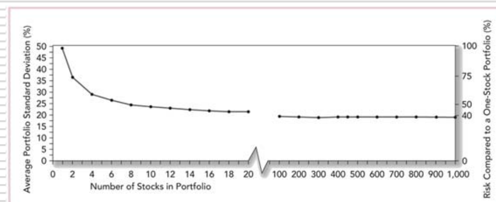
Some risk factors (e.g. global recessions) affect all companies in the economy
→ this is called market risk, systematic risk, aggregate risk, or undiversifiable risk

Each stock is also subject to firm-specific risk factors
→ this is also called unique risk, nonsystematic risk or diversifiable risk

Diversification: by investing into many different firms, firm-specific risk can be reduced

("not putting all your eggs into one basket")

Diversification: An Illustration



Portfolios of Two Risky Assets

Assume a portfolio P of two risky assets D and E with portfolio weights w_D and w_E

Return $r_P = w_D r_D + w_E r_E$

$E[r_P] = w_D E[r_D] + w_E E[r_E]$

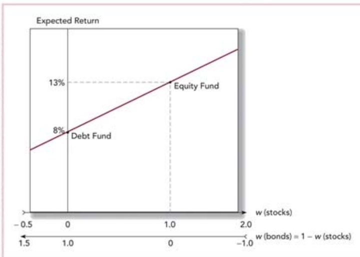
$\sigma_P^2 = w_D^2 \sigma_D^2 + w_E^2 \sigma_E^2 + 2 w_E w_D Cov(r_D, r_E)$

recall: $Cov(r_D, r_E) = \rho_{DE} \sigma_D \sigma_E$ where $-1 \le \rho_{DE} \le 1$

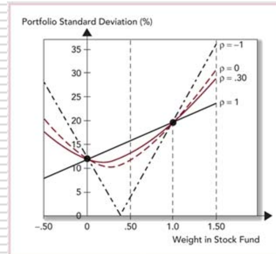
Example of Two Risky Assets

	Debt	Equity
Expected return, $E(r)$	8%	13%
Standard deviation, σ	12%	20%
Covariance, $Cov(r_D, r_E)$	72	
Correlation coefficient, ρ_{DE}	.30	

Expected Return Two Risky Assets



Standard Deviation Two Risky Assets



Minimum Variance Portfolio

What is the lowest variance that we can obtain per dollar invested using D and E?

→ set $w_E = 1 - w_D$

→ min $w_D^2\sigma_D^2 + (1-w_D)^2\sigma_E^2 + 2(1-w_D)w_D Cov(r_D,r_E)$

- if $\rho_{DE} = 1$: no diversification possible
- if $-1 < \rho_{DE} < 1$: some diversification possible
- if $\rho_{DE} = -1$: full diversification can be obtained

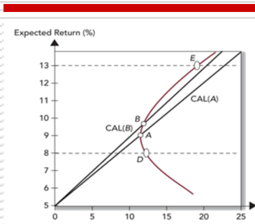
Asset Allocation with Stocks and Bonds

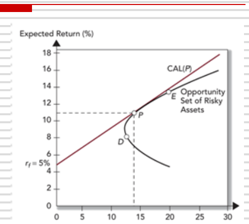
Last lecture: capital allocation problem of how to allocate investments between risk-free and risky portfolio

This lecture: asset allocation problem of how to allocate investment between different risky assets

Combination of the two problems: asset allocation between risk-free and two risky assets

Asset Allocation: Examples





We can obtain different Capital Allocation Lines between the risk-free asset and various risky portfolios

Asset Allocation with Stocks and Bonds

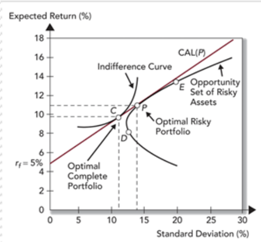
1) Steepest CAL offers most attractive set of investment opportunities

→ maximize slope (= Sharpe ratio): max $S_P = (E[r_P] - r_f)/\sigma_P$

→ determine optimal risky portfolio

2) Determine optimal complete portfolio, as discussed in previous lecture

Optimal Complete Portfolio



Generalization: Multiple Risky Assets

Markowitz Portfolio Selection Model:

1) Determine the minimum-variance frontier of all risky assets
2) Upper part of the MV-frontier = efficient frontier of risky assets
3) Pick the highest possible CAL
   → determine optimal risky portfolio
4) Determine optimal complete portfolio

Separation property

Separation property =
we can determine the optimal risky and the optimal complete portfolio independently

→ Investors with different risk aversion will pick the same risky portfolio, but will put different weights on it