Lecture 7: Dealing with Risk

ECON435: Financial Markets and the Macroeconomy

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Review: Risk and Return

- Real returns vs. nominal returns
- Risk-free interest rates
- EAR vs. APR
- Holding period return
- Risk premium
- Sharpe ratio
- Value-at-Risk

Speculation vs. Gambling

Speculation = taking on considerable risk to obtain a commensurate gain

Gambling = taking on risk for enjoyment

Fair game: expected return = 0

Risk Aversion and Investor Utility

Risk averse investors demand a risk premium (excess return) as compensation for taking on risk

We can capture investors' risk/return preferences using a utility function:

$$U = E(r) - \tfrac{1}{2}A\sigma^2$$

$E(r)$ expected return of investment
A index of risk aversion
σ^2 variance of investment

Calculation of Utility Scores

Portfolio	Risk Premium	Expected Return	Risk (SD)
L (low risk)	2%	7%	5%
M (medium risk)	4	9	10
H (high risk)	8	13	20

Example of three different portfolios

Investor Risk Aversion (*A*)	Utility Score of Portfolio *L* [$E(r) = .07; \sigma = .05$]	Utility Score of Portfolio *M* [$E(r) = .09; \sigma = .10$]	Utility Score of Portfolio *H* [$E(r) = .13; \sigma = .20$]
2.0	$.07 - ½ \times 2 \times .05^2 = .0675$	$.09 - ½ \times 2 \times .1^2 = .0800$	**$.13 - ½ \times 2 \times .2^2 = .09$**
3.5	$.07 - ½ \times 3.5 \times .05^2 = .0656$	**$.09 - ½ \times 3.5 \times .1^2 = .0725$**	$.13 - ½ \times 3.5 \times .2^2 = .06$
5.0	$.07 - ½ \times 5 \times .05^2 = .0638$	**$.09 - ½ \times 5 \times .1^2 = .0650$**	$.13 - ½ \times 5 \times .2^2 = .03$

Corresponding utility scores for different A's

Interpreting Utility Scores

Utility score of risky investment = return on certain investment that would make investor indifferent

$A > 0$: risk-averse investor (normal case)
$A = 0$: risk-neutral investor
$A < 0$: risk-loving investor

Mean-Variance (M-V) Criterion

Portfolio A dominates portfolio B if

$$E(r_A) \geq E(r_B) \quad \text{and} \quad \sigma_A^2 \leq \sigma_B^2$$

with at least one inequality holding strictly

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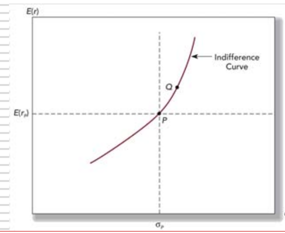
Indifference Curves

Indifference curves connect all points in a graph of E(r) versus σ with the same utility



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Numerical Derivation of Indifference Curves

Find all combinations of E(r) and σ^2 that yield the same utility score:

Expected Return, $E(r)$	Standard Deviation, σ	Utility = $E(r) - ½ A\sigma^2$
.10	.200	.10 − .5 × 4 × .04 = .02
.15	.255	.15 − .5 × 4 × .065 = .02
.20	.300	.20 − .5 × 4 × .09 = .02
.25	.339	.25 − .5 × 4 × .115 = .02

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The Asset Allocation Problem

Most fundamental decision in asset allocation problem =

= choice of how much to invest in

- risky assets: stocks, bonds, …
- versus risk-free assets: T-bills, …

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Portfolios with one Risky Asset

Assume we have a choice between:

- a risky portfolio with expected return r_P and standard deviation σ_P
- a risk-free asset with return r_f

If we invest a fraction y of our wealth in the risky portfolio, the return of the resulting **complete portfolio** is:

$r_C = yr_P + (1-y)r_f = r_f + y(r_P-r_f)$

The expected return and the standard deviation are:

$E[r_C] = yE[r_P] + (1-y)r_f = r_f + y(E[r_P]-r_f)$

$\sigma_C = y\sigma_P \rightarrow y = \sigma_C/\sigma_P$

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The Capital Allocation Line (CAL)

We can express the return $E[r_C]$ as a function of σ_C:

$$E[r_C] = r_f + \sigma_C*(E[r_P]-r_f)/\sigma_P$$

Note: the last term is the Sharpe ratio; it captures the reward-to-volatility ratio

→ plotting this function in a graph of E[r] versus σ yields the **Capital Allocation Line**

→ this line captures the opportunity set of potential investments

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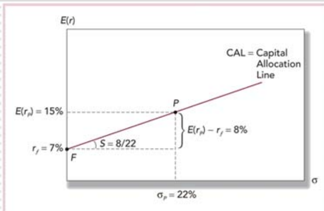
Example of Capital Allocation Line



Optimal Portfolio Choice

Investor chooses a fraction y to maximize the utility of her complete portfolio:

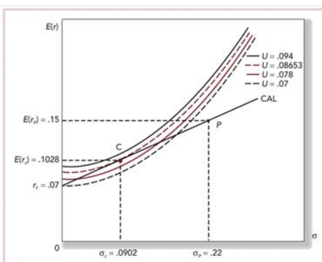
$\max_{(y)} U = E[r_C] - ½A\sigma_C^2$
$= r_f + y(E[r_P] - r_f) - ½Ay^2\sigma_P^2$

$y^* = (E[r_P] - r_f)/(A\sigma_P^2)$

Invest more in the risky portfolio P:

- the greater the expected return $E[r_P]$
- the smaller the variance σ_P^2
- the smaller the investor's risk aversion

Graphic: Optimal Portfolio Choice



Capital Market Line

Many investors follow a "passive" investment strategy, i.e. they invest into a broad market index

→ the risky portfolio represents the entire capital market, e.g. S&P500, …

→ the capital allocation line between 1-month T-bills and a broad market index is called the
capital market line