# Lecture 13: Term Structure of Interest Rates

ECON435: Financial Markets and the Macroeconomy

Anton Korinek

Spring 2011



## Review: Bond Prices and Yields

Bond price = PV(coupons) + PV(par value):

$$P_B = \sum_{t=1}^{T} \frac{C}{(1+r)^t} + \frac{ParValue}{(1+r)^T}$$

Convexity = inverse relationship between bond prices and yields

Yield to Maturity (YTM) = interest rate at which a bond's PV equals its market price



## Term Structure

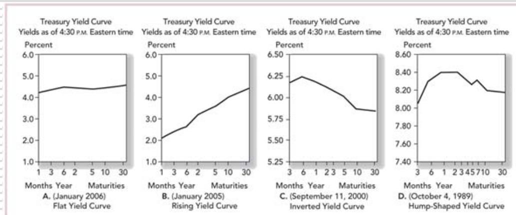
Last lecture: we assumed that bond yields (discount rates) were constant over time

In fact: bond yields vary with maturity

→ Yield curve: depicts bond yields as a function of maturity



## Different Treasury Yield Curves




flat rising inverted hump-shaped

→ compare to today's yield curve



## Bond Pricing using the Term Structure

Each bond payment needs to be discounted at appropriate rate

Example 1: Zero coupon bonds under a rising yield curve

| Maturity (years) | Yield to Maturity (%) | Price |
|---|---|---|
| 1 | 5% | $952.38 = $1,000/1.05 |
| 2 | 6 | $890.00 = $1,000/1.06$^2$ |
| 3 | 7 | $816.30 = $1,000/1.07$^3$ |
| 4 | 8 | $735.03 = $1,000/1.08$^4$ |



## Bond Pricing using the Term Structure

Example 2: Value a two-year bond with 8% coupon and $100 face value using the yields from example 1

Year 1: $8 coupon → $8/1.05 =

Year 2: $108 coupon + principal → $108/1.06$^2$ =

Total value =

## Future Short-Term Rates

Two-year yield $y_2$ = [geometric] average of:

- current 1-year yield $r_1$
- future 1-year interst rate $r_2$ between periods 1 and 2 (assuming no uncertainty)

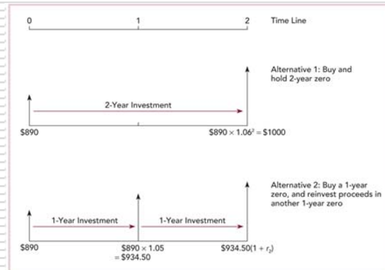
$$(1+y_2)^2 = (1+r_1)\cdot(1+r_2)$$

or

$$1+y_2 = \sqrt{(1+r_1)\cdot(1+r_2)}$$



## Future Short-Term Rates






## Forward Rates

Definition of "forward interest rate" =

= expected future short-term interest rate

For any investment horizon n:

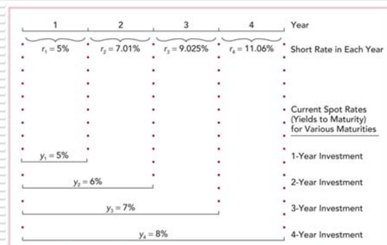
$$(1+y_n)^n = (1+y_{n-1})^{n-1}\cdot(1+f_n)$$

or

$$1+f_n = \frac{(1+y_n)^n}{(1+y_{n-1})^{n-1}}$$



## Forward Rates






## Forward Rates: Example

Example: $y_3$ = 7%, $y_4$ = 8%

$$1+f_4 = \frac{(1+y_4)^4}{(1+y_3)^3} = \frac{(1.08)^4}{(1.07)^3} = 1.1106$$

→ $f_4$ = 11.06%



## Slope of the Yield Curve and Forward Rates

If yield curve

- rising: forward rates > short rate
- flat: forward rates = short rate
- inverted: forward rates < short rate

This is because:

- spot yield = average of current short rate and forward rates

## Theories of the Term Structure

1. Expectations Hypothesis:
   - short-term and long-term bonds are perfect substitutes
   - forward rates reflect market expectations of future short-term rates:

     $$f_n = E[r_n]$$

   - long-term yields are a product of short-term and expected future short-term rates:

     $$(1+y_n)^n = (1+r_1)\cdot(1+E[r_2])\cdot \ldots \cdot(1+E[r_n])$$

13

## Theories of the Term Structure

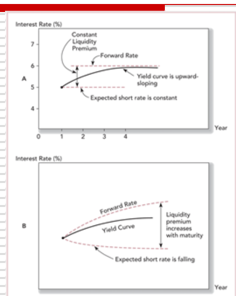
2. Liquidity Preference Theory:
   - long-term bonds are more risky
   - investors demand a liquidity risk premium
   - this raises forward rates above expected future short rates:

     $$f_n > E[r_n]$$

   - the yield curve has an upward-bias

14

## Liquidity Theory: Illustration




15

## Liquidity Theory: Example

- Assume $r_1 = 5\%$, $y_2 = 6\%$, $y_3 = 7\%$ and $E[r_3] = 7\%$
- What is the liquidity premium on three-year forwards?

$1+f_3 = (1+y_3)^3/(1+y_2)^2 = 1.07^3/1.06^2 =$

Liquidity premium $= f_3 - E[r_3] =$

16